The raw number of new infections in a given year is

$$\text{New infections[n]} = \beta[n] * \frac{I[n]}{N[n]} * S[n]$$

where:

- $S[n]$ = number of susceptible in given year
- $I[n]$ = number of infected in given year
- $N[n]$ = total population in given year
- $\beta[n]$ = effective contact rate

But the data I have (from UNAIDS) gives $HIV_{Incidence}$ in "per 1000" so:

$$HIV_{Incidence}[n] = \text{New infections[n]} * \frac{1000}{N[n]}$$

$$\iff HIV_{Incidence}[n] = \beta[n] * \frac{I[n]}{N[n]} * \frac{S[n]}{N[n]} * 1000$$

- $\frac{I[n]}{N[n]} = HIV_{Prevalence}$ is the prevalence that I have from UNAIDS as a proportion (from 0 to 1)
- $\frac{S[n]}{N[n]} = 1 - \frac{I[n]}{N[n]} = 1 - HIV_{Prevalence}$ (same from 0 to 1 depending on above)

I let β absorb the factor of 1000 and I end up with:

$$\beta = \frac{HIV_{Incidence}}{HIV_{Prevalence} * (1 - HIV_{Prevalence})}$$

Example:

2 countries with same $\beta = 10^{-3}$ and $HIV_{Prevalence} = 20\% = 0.2$ but different total populations:

- Country A has population $N_A = 10^9$
- Country B has population $N_B = 10^5$

New infections:

- Country A then has $10^{-3} * 0.2 * 0.8 * 10^9 = 1.6 * 10^5$ new infections
- Country B then has $10^{-3} * 0.2 * 0.8 * 10^5 = 16$ new infections

Incidence in "per 1000":

- Country A has $1.6 * 10^5 * \frac{10^3}{10^9} = 0.16$
- Country B has $1.6 * 10^1 * \frac{10^3}{10^5} = 0.16$